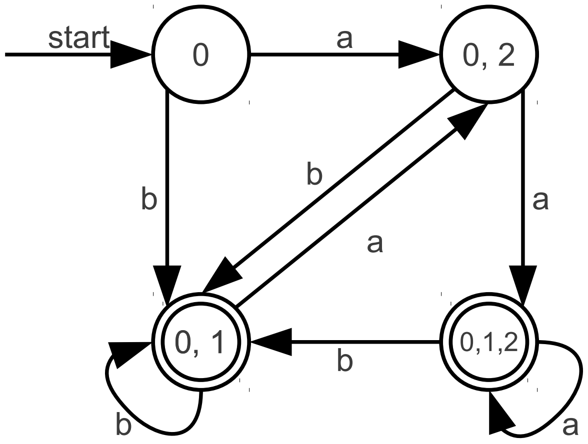
CS143
Summer 2012

Handout 07S
July 13, 2012

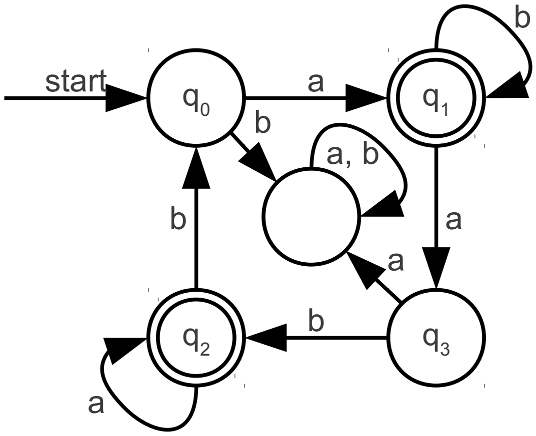
Written Set 1 Solutions

Problem One: Subset Construction

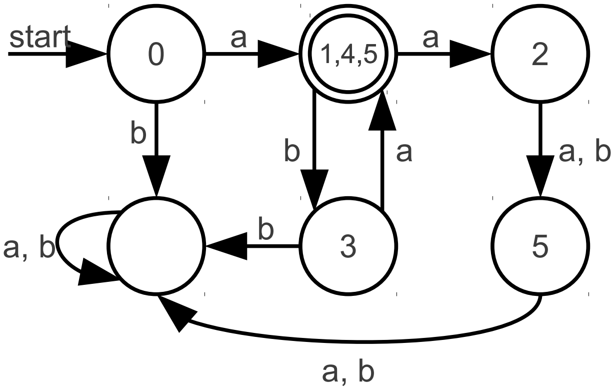
i.



ii.



iii.



Problem Two: Maximal Munch

```
%%
a*b             printf("1");
(a|b)*b         printf("2");
c*              printf("3");
```

i. aaabccabbb

The result is **`132`**, with the tokenization aaab cc abbb.

ii. cbbbbac

The result is **`32a3`**. The tokenization is c bbbb a c, where the single character 'a' does not match any regular expression and is thus echoed back to the console.

iii. cbabc

The result is **`323`**, with tokenization c bab c.

Problem Three: The Limits of Conflict Resolution

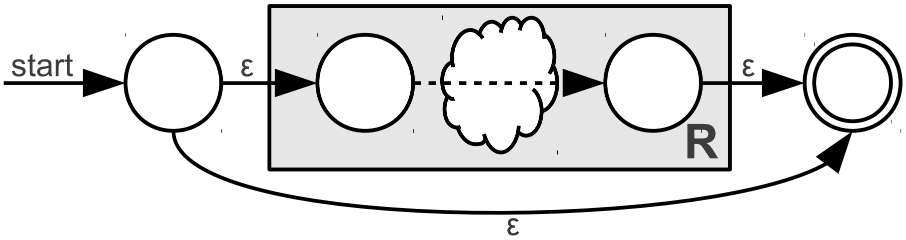
Consider this flex script:

```
%%
"aa"                    { return 1; }
"a"                     { return 2; }
"ab"                    { return 3; }
```

The string "aab" could be tokenized as "a," "ab." However, maximal-munch will first match "aa," and then will fail to match "b."

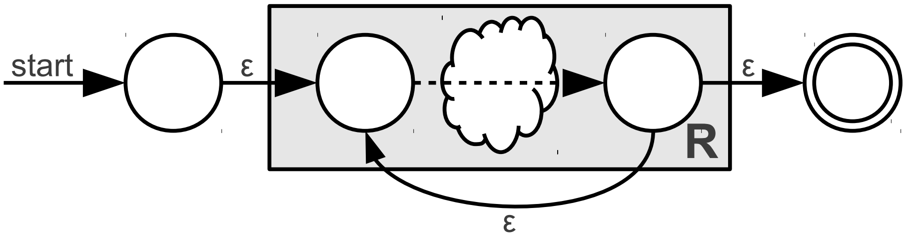
Problem Four: Converting Extended Regular Expressions

1. **R?**, which matches zero or one copies of **R**:



Intuitively, we can either skip over the machine for **R**, or work through the machine.

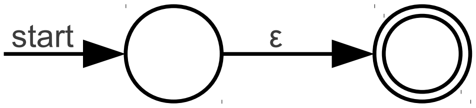
2. **R+**, which matches one or more copies of **R**:



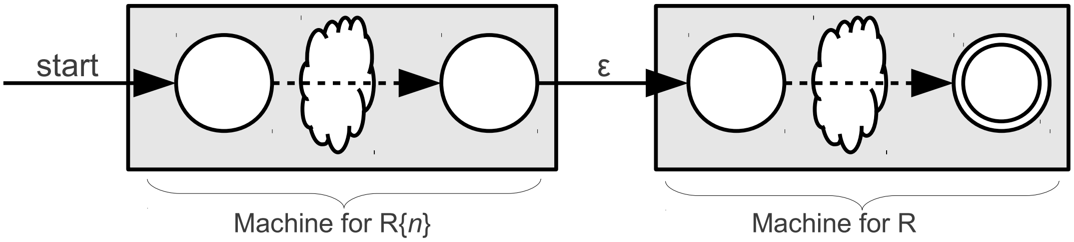
Intuitively, we have to make it through the machine for **R** at least once, and can then cycle around through it as many more times we'd like.

3. **R{n}**, which matches exactly *n* copies of **R**.

   This construction is defined inductively. We match **R**{0} with



   Then, we will match **R**{*n* + 1} with



This works because we can inductively define **R**{0} = ε, and **R**{*n* + 1} = **RR**{*n*}.

Problem Five: Right-to-Left Scanning

i. Modify the existing algorithm for converting regular expressions to NFAs so that the generated NFA accepts the **reverse** of strings that match the regular expression. Briefly justify why your construction is correct.

There are *many* approaches to solving this problem. Here are three:

1. You can construct the NFA as before, then reverse all of the transition arrows. Then, make the old accept state a new start state, and the old start state the new accept state.
2. You can transform the regular expression as follows, and then apply the existing algorithm: given a regular expression R, define the function REV as follows:
    1. REV(a) = a for any single character a,
    2. REV(ε) = ε for any single character ε,
    3. REV(R_1 | R_2) = REV(R_1) | REV(R_2),
    4. REV(R_1 R_2) = REV(R_2) REV(R_1),
    5. REV(R*) = REV(R)*, and
    6. REV((R)) = (REV(R))

    For example, REV(a(b|c)*d) = d(b | c)*a.
3. You can modify the construction for the R_1R_2 portion of the construction so that instead of chaining R_1 into R_2, instead you chain R_2 into R_1, so that the contents of R_2 are matched before R_1.

ii. Give an example of a set of regular expressions and a string so that the left-to-right scan of the string produces a different set of tokens than the right-to-left scan. Assume that you're using the maximal-munch algorithm for conflict resolution.

Here is one possible set of regular expression:

```
%%
aa                   { return 1; }
ab                   { return 1; }
a|b                  { return 2; }
```

If you scan the string aab from left-to-right, you get the tokenization aa b. If you scan this string from right to left, you get a ab.

Problem Six: Slowing Down `flex` Scanners

Consider the following **`flex`** script:

```
%%
a*b                 { return 1; }
a                   { return 2; }
```

Then let $f(n) = \mathrm{a}^n$ (that is, n copies of the character a). When the above scanner runs on this string, it will have to scan all n characters on the first iteration to check to see that the regular expression a*b does not match. Since it does not, it will use the second regular expression to match just the first character. The next iteration will scan all remaining $n - 1$ characters before matching just one a, the iteration after that will scan $n - 2$ characters, etc. This means that the number of characters scanned is

$$n + (n - 1) + (n - 2) + \ldots + 2 + 1 = n(n + 1) \; / \; 2$$

which is $\Theta(n^2)$.